

Bose-Einstein correlations and the transition time from QCD plasma to hadrons

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Abstract

It is pointed out that the size of the interaction region, as determined from HBT analyses, is increased due to the transition time necessary to convert the quarks and gluons into hadrons. A rough estimate yields an increase of R_{HBT} by about 15%.

1. It is now well established that particle production in high energy heavy ion collisions proceeds through the intermediate state of a quark-gluon plasma. Much effort is devoted to investigate the properties of this state and of the transition from the plasma to the observed hadrons. In these investigations measurements of quantum interference (HBT) play a particularly interesting role, being one of the very few sources of information about the space-time properties of the system.

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The difficulty is that neither the properties of the plasma, nor the mechanism of hadron formation are sufficiently well known. One point, however, is non-controversial. This transition must take a certain time which we call the transition time.

One can invoke several origins for the transition time. The hadrons do not interact before they are dressed. This requires a certain time known as the formation time. Effects of the formation time on the HBT measurements have been recently discussed in the framework of UrQMD [1]¹. Even dressed constituents cannot form a hadron instantly, however. Stodolsky [3] analyzed from this point of view the formation of the hydrogen atom from a proton and an electron. and adapted this argument for the $q\bar{q}$ system. His reasoning is briefly presented as the second comment towards the end of the present note. Moreover, in order to produce a white hadron from a coloured plasma, it is necessary either to rearrange the plasma constituents into white lumps (preconfinement), or to get rid of the unwanted colour by emitting one, or more, soft gluons. This list could be made longer.

The consequences of the non-vanishing transition time are twofold. First, it increases the time after which the observed hadrons appear, as compared to estimates following, e.g., from hydrodynamics. Second, since the plasma created in heavy ion collisions is far from static, the formation time leads to an increase of the measured volume from which the hadrons emerge.

Thus it seems interesting to investigate how the presence of the formation time changes the space-time structure of the system and how it influences the results of the HBT measurements. This is the purpose of the present note in which we shall analyze these phenomena in a simplified model.

2. We consider the two-dimensional problem (in the transverse plane), assuming -for simplicity- uncorrelated particle production². Denoting the single-particle Wigner function by $W(p, x)$, the observed single- and two-identical-particle momentum distributions are given by

$$\Omega_0(p) = \int d^2x W(p, x); \quad (1)$$

$$\Omega(p_1, p_2) = F(p_1)F(p_2) [1 + C(p_1, p_2)] \quad (2)$$

¹The connection of the formation time to the uncertainty principle was considered in [2].

²Corrections following from interparticle correlations are discussed, e.g., in [4]

where

$$C(p_1, p_2) = \frac{|H(Q, K)|^2}{F(p_1)F(p_2)} \quad (3)$$

is the correlation function and

$$H(Q, K) = \int d^2x e^{iQx} W(K, x); \quad F(p) = H(Q = 0, p) = \int d^2x W(p, x) \quad (4)$$

with $K = (p_1 + p_2)/2$ and $Q = p_1 - p_2$.

3. Our approach is purely phenomenological. Consider a final hadron. It is formed from quarks and gluons emerging from a relatively small "lump" of plasma (they have to be close enough in space to form a hadron). Following the argument presented before, we assume that in the rest frame of the lump this process takes a certain time τ_0 . Naturally, this time may depend on the kind of the hadron. In the system where the lump of plasma moves with velocity v , τ_0 is multiplied by the corresponding Lorentz factor: $\tau = \gamma\tau_0$.

Consider now the distribution of plasma at freeze-out, $G[\bar{x}]d^2\bar{x}$ (e.g. the one calculated from hydro), in the Lorentz frame where the longitudinal momentum of the considered piece of plasma vanishes (LCMS frame). It gives the distribution of the fluid in the transverse plane (\bar{x} denotes the transverse position vector in the plasma³). At each \bar{x} also the velocity of plasma $v = v(\bar{x})$ is known. E.g., in the simple case of the Hubble flow we have

$$v(\bar{x})\gamma(\bar{x}) = \mu\bar{x} \quad (5)$$

where μ is a constant.

It should be emphasized that $v(\bar{x})$ is the velocity of the lump of plasma located at position \bar{x} and it is not identical with the velocities of the produced hadrons. Indeed, in the rest frame of the lump, the momenta \hat{p} of the particles, though equal zero on the average, have some spherically symmetric distribution. For instance, in the statistical model they follow the Boltzmann distribution. Therefore, the actual distribution is

$$W[\bar{x}, p]d^2\bar{x}d^2p = G[\bar{x}]\hat{U}[\bar{x}; \hat{p}]d^2\bar{x}d^2\hat{p}/\hat{E} = G[\bar{x}]U[\bar{x}; p]d^2\bar{x}d^2p/E \quad (6)$$

³As already mentioned, all vectors are two-dimensional and lie in the transverse plane of the collision

where \hat{U} is the distribution of hadron momenta in the rest frame of the lump, and U is the same distribution expressed in terms of the momenta in the laboratory system, where the lump moves with velocity $v(\bar{x})$. The momenta are of course related by the Lorentz transformation:

$$\hat{p}_{\parallel} = \gamma\{p_{\parallel} - vE\}; \quad \hat{E} = \gamma\{E - vp_{\parallel}\}; \quad E = \sqrt{p^2 + m^2}; \quad \hat{p}_{\perp} = p_{\perp} \quad (7)$$

where the subscripts \parallel and \perp refer to the direction of $v(\bar{x})$.

4. In the rest frame of the lump, a hadron is emitted at some time τ_0 which of course need not be the same for all hadrons. In the laboratory system, this time is multiplied by $\gamma(\bar{x})$. During the time $\tau = \gamma(\bar{x})\tau_0$ the lump moves in the laboratory system from the position \bar{x} to the position

$$x = \bar{x} + \Delta\bar{x} = \bar{x} + v(\bar{x})\gamma(\bar{x})\tau_0. \quad (8)$$

This formula allows to express \bar{x} in terms of x and τ_0 . Consequently, the distribution of the emission points becomes

$$W[x, p]d^2x d^2p/E = G[x - \Delta\bar{x}]U[\bar{x}; p] \left| \frac{d^2\bar{x}}{d^2x} \right| d^2x d^2p/E \quad (9)$$

where $|d^2\bar{x}/d^2x|$ is the Jacobian of the change of variables from \bar{x} to x .

To see the physical consequences of (9) we have to evaluate the function $H(Q, K)$ defined in (4). We have

$$H(Q, K) = \int d^2x e^{iQx} G[x - \Delta\bar{x}]U[\bar{x}; K] \left| \frac{d^2\bar{x}}{d^2x} \right| / E_K \quad (10)$$

where $E_K = \sqrt{K^2 + m^2}$. Changing the integration variable from x back to \bar{x} we obtain

$$H(Q, K) = \int d^2\bar{x} e^{iQ[\bar{x} + \Delta\bar{x}]} G[\bar{x}]U[\bar{x}; K] / E_K. \quad (11)$$

One sees from this formula that the dependence of the result on τ_0 comes only from the term $\Delta\bar{x} = v(\bar{x})\gamma(\bar{x})\tau_0$, present in the exponent and multiplied by Q . From this observation one sees immediately that there is no effect of the formation time when there is no transverse flow, i.e. for a static plasma. One also sees that the formation time does not change the single particle momentum distributions.

5. A particularly simple situation is obtained in the case of a radial transverse flow, i.e. when the velocity $v(\bar{x})$ points in the same direction as \bar{x} (this is the case in most hydrodynamical models⁴). Then we can write

$$v(\bar{x}) = |v(\bar{x})| \frac{\bar{x}}{|\bar{x}|} \quad (12)$$

and thus

$$H(Q, K) = \int d\bar{x} e^{iQ_{eff}\bar{x}} G[\bar{x}] U[\bar{x}; K] / E_K, \quad (13)$$

where

$$Q_{eff} = Q \left[1 + \frac{|v(\bar{x})|}{|\bar{x}|} \gamma(\bar{x}) \tau_0 \right]. \quad (14)$$

This observation implies that the HBT radii obtained from (13) differ from those obtained from the hydrodynamical calculations by the factor $1 + [|v(\bar{x})|/|\bar{x}|] \gamma(\bar{x}) \tau_0$ averaged over the plasma distribution.

In case of the simple Hubble flow (5) we obtain

$$Q_{eff} = Q[1 + \mu \tau_0] \quad (15)$$

and thus we conclude that in this case the observed *HBT* radii should be larger than those evaluated from hydrodynamics by the factor $[1 + \mu \tau_0]$. Note that this result is independent of the distribution of plasma in the transverse plane.

To obtain an estimate of the size of the enhancement factor, one needs the value of the parameter μ , responsible for the transverse flow. It seems reasonable to take $\mu \approx 0.1/\text{fm}$, giving $v\gamma = 1$ at $|\bar{x}| = R = 10 \text{ fm}$, in agreement with the arguments of [6, 7]. Then, taking $\tau_0 = 1/m_\pi$, one finds a correction of about 15%.

To estimate the sensitivity of this result on the form of the transverse flow we also considered the relation suggested in [8]

$$|v(\bar{x})| \gamma(\bar{x}) = \sinh(\mu_1 \bar{x}). \quad (16)$$

The results were evaluated numerically, using

$$G(\bar{x}) = e^{-\bar{x}^2/R^2}; \quad U(\bar{x}, p) = e^{-\hat{E}/T} = e^{-\gamma(\bar{x})[E - |v(\bar{x})|p_\parallel]/T} \quad (17)$$

⁴See, however, [5]

with $R = 10$ fm, $T = 160$ MeV. The parameter μ_1 was taken 0.08/fm, adjusted to obtain the same radii as in case of Hubble flow for $\tau_0 = 0$ and $\mu = 0.1$. This exercise showed that the effects of the transition time on R_{HBT}^{side} are similar to those due to the Hubble flow (the result (15) is not changed by more than a few percent). The increase of R_{HBT}^{out} is similar, though somewhat larger. We thus conclude that, within an error of a few percent, the effect of the formation time is not sensitive to the form of the transverse flow. Furthermore, this exercise shows that, at least for the transition temperature in the region of 160 MeV, it is also not substantially affected by the correlation between $v(\bar{x})$ and K , induced by the function $U[\bar{x}, K]$ and by the dependence of γ and v on \bar{x} (see (7)).

6. In conclusion, using a simplified two-dimensional model, we have investigated the effects of the formation time of hadrons on the results of the HBT measurements. It was shown that the HBT radius measured from $\pi\pi$ correlations is larger, at least by about 15%, than that resulting from the hydrodynamical evolution.

Some comments are in order.

(i) Our argument is based on a simplified, two-dimensional model. Therefore it applies, strictly speaking, only to the case where the longitudinal momenta of both particles are identical (and thus both vanish in the LCMS). The corrections related to the longitudinal motion tend to decrease the effect of the transition time. They are not very important, however, if the momentum difference is kept below the pion mass.

(ii) The transition time we are discussing in this paper should not be identified with the hadron formation time estimated traditionally to be $\sim 1/m$. First, as suggested in the introduction, there may be other processes contributing to it. Second, it should be remembered that $\tau_0 = 1/m$ is only the smallest possible value consistent with the uncertainty principle. It was observed in [3] that one can construct a classical argument which points out to a hadron formation time which is larger by a significant factor. Consider a particle as a bound state of two (constituent) quarks. It seems reasonable to accept that this state can be considered as "formed" when the constituents make at least one turn around each other. Then one finds $\tau_0 \approx 2\pi r/v$ where r is the radius of the particle and v is the velocity on the orbit. Taking $r \approx 1$ fm, and putting for v its upper limit $v = 1$, one obtains $\tau_0 \approx 2\pi$ fm $\approx 4.5/m_\pi$. Note that this argument suggests that the actual value of the formation time may be related to the *size* of the produced hadron rather than to its mass.

(iii) The purpose of this note is only to point out the possible effects of the formation time but not their precise estimate. A similar but technically much more detailed analysis of the role of the time delay in hadron production can be found in [8]. Although the discussion in this paper refers to the production of the long-living resonances, their results may be useful also in the quantitative studies of the formation time effects.

(v) The transition time we are discussing here is the time needed to form well defined hadrons from the quark-gluon plasma. Collisions between the produced hadrons, which may further influence the measurements of the HBT radii, are of course possible and even expected [9]. But this problem goes beyond the scope of the present note.

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